

Table 17.8

HEUMAN'S LAMBDA FUNCTION $\Lambda_0(\varphi|\alpha)$

$$\Lambda_0(\varphi|\alpha) = \frac{F(\varphi|90^\circ - \alpha)}{K'(\alpha)} + \frac{2}{\pi} K(\alpha) Z(\varphi|90^\circ - \alpha) = \frac{2}{\pi} \{K(\alpha) E(\varphi|90^\circ - \alpha) - [K(\alpha) - E(\alpha)] F(\varphi|90^\circ - \alpha)\}$$

$\alpha \backslash \varphi$	0°	5°	10°	15°	20°	25°	30°
0°	0	0.087156	0.173648	0.258819	0.342020	0.422618	0.500000
2	0	0.087129	0.173595	0.258740	0.341916	0.422490	0.499848
4	0	0.087050	0.173437	0.258504	0.341604	0.422104	0.499391
6	0	0.086917	0.173173	0.258111	0.341084	0.421462	0.498633
8	0	0.086732	0.172804	0.257562	0.340359	0.420566	0.497574
10	0	0.086495	0.172332	0.256858	0.339430	0.419419	0.496219
12	0	0.086206	0.171757	0.256001	0.338299	0.418024	0.494572
14	0	0.085866	0.171080	0.254994	0.336969	0.416385	0.492638
16	0	0.085476	0.170303	0.253838	0.335445	0.414506	0.490424
18	0	0.085037	0.169429	0.252536	0.333729	0.412394	0.487937
20	0	0.084549	0.168458	0.251092	0.331827	0.410054	0.485184
22	0	0.084013	0.167393	0.249509	0.329743	0.407492	0.482176
24	0	0.083432	0.166236	0.247790	0.327483	0.404717	0.478920
26	0	0.082806	0.164991	0.245941	0.325052	0.401736	0.475428
28	0	0.082136	0.163661	0.243966	0.322458	0.398558	0.471710
30	0	0.081425	0.162247	0.241870	0.319707	0.395191	0.467777
32	0	0.080674	0.160755	0.239657	0.316806	0.391645	0.463642
34	0	0.079884	0.159187	0.237335	0.313764	0.387930	0.459316
36	0	0.079058	0.157548	0.234908	0.310587	0.384057	0.454813
38	0	0.078198	0.155842	0.232383	0.307286	0.380037	0.450147
40	0	0.077307	0.154073	0.229767	0.303869	0.375880	0.445330
42	0	0.076385	0.152246	0.227068	0.300346	0.371600	0.440378
44	0	0.075436	0.150367	0.224292	0.296727	0.367209	0.435306
46	0	0.074463	0.148439	0.221447	0.293022	0.362720	0.430127
48	0	0.073469	0.146470	0.218543	0.289242	0.358145	0.424860
50	0	0.072455	0.144464	0.215587	0.285399	0.353500	0.419519
52	0	0.071426	0.142428	0.212589	0.281505	0.348799	0.414121
54	0	0.070385	0.140370	0.209558	0.277573	0.344057	0.408685
56	0	0.069336	0.138295	0.206506	0.273616	0.339290	0.403228
58	0	0.068281	0.136211	0.203443	0.269648	0.334516	0.397769
60	0	0.067226	0.134126	0.200380	0.265684	0.329751	0.392328
62	0	0.066175	0.132049	0.197331	0.261739	0.325015	0.386926
64	0	0.065131	0.129989	0.194307	0.257832	0.320328	0.381586
66	0	0.064100	0.127955	0.191324	0.253979	0.315710	0.376331
68	0	0.063088	0.125958	0.188396	0.250200	0.311185	0.371186
70	0	0.062100	0.124009	0.185540	0.246517	0.306778	0.366180
72	0	0.061143	0.122121	0.182774	0.242952	0.302515	0.361342
74	0	0.060223	0.120307	0.180119	0.239531	0.298427	0.356706
76	0	0.059348	0.118583	0.177596	0.236282	0.294547	0.352309
78	0	0.058528	0.116967	0.175231	0.233238	0.290914	0.348194
80	0	0.057773	0.115479	0.173054	0.230436	0.287571	0.344410
82	0	0.057095	0.114143	0.171099	0.227922	0.284573	0.341017
84	0	0.056508	0.112988	0.169410	0.225750	0.281983	0.338088
86	0	0.056034	0.112053	0.168043	0.223992	0.279887	0.335718
88	0	0.055698	0.111392	0.167078	0.222751	0.278408	0.334046
90	0	0.055556	0.111111	0.166667	0.222222	0.277778	0.333333
		$\left[\begin{smallmatrix} (-5)2 \\ 5 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)5 \\ 5 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)7 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-5)9 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$
5	0	0.086990	0.173318	0.258327	0.341370	0.421815	0.499050
15	0	0.085677	0.170704	0.254434	0.336231	0.415475	0.491565
25	0	0.083124	0.165625	0.246882	0.326288	0.403252	0.477203
35	0	0.079476	0.158377	0.236134	0.312192	0.386013	0.457086
45	0	0.074953	0.149408	0.222878	0.294884	0.364976	0.432729
55	0	0.069861	0.139334	0.208034	0.275597	0.341676	0.405958
65	0	0.064614	0.128968	0.192809	0.255897	0.318009	0.378946
75	0	0.059779	0.119433	0.178839	0.237883	0.296459	0.354475
85	0	0.056256	0.112490	0.168682	0.224814	0.280867	0.336826

Compiled from C. Heuman, Tables of complete elliptic integrals, J. Math. Phys. 20, 127-206, 1941 (with permission).

HEUMAN'S LAMBDA FUNCTION $\Lambda_0(\varphi|\alpha)$

Table 17.8

$$\Lambda_0(\varphi|\alpha) = \frac{F(\varphi|90^\circ - \alpha)}{K'(\alpha)} + \frac{2}{\pi} K(\alpha) Z(\varphi|90^\circ - \alpha) = \frac{2}{\pi} \{K(\alpha) E(\varphi|90^\circ - \alpha) - [K(\alpha) - E(\alpha)] F(\varphi|90^\circ - \alpha)\}$$

$\alpha \backslash \varphi$	35°	40°	45°	50°	55°	60°
0°	0.573576	0.642788	0.707107	0.766044	0.819152	0.866025
2	0.573402	0.642592	0.706891	0.765811	0.818903	0.865762
4	0.572878	0.642006	0.706247	0.765113	0.818157	0.864975
6	0.572009	0.641032	0.705177	0.763956	0.816922	0.863674
8	0.570795	0.639674	0.703687	0.762347	0.815210	0.861876
10	0.569244	0.637940	0.701786	0.760298	0.813034	0.859602
12	0.567360	0.635836	0.699484	0.757822	0.810416	0.856877
14	0.565150	0.633373	0.696794	0.754937	0.807375	0.853731
16	0.562623	0.630561	0.693729	0.751660	0.803935	0.850194
18	0.559789	0.627412	0.690306	0.748011	0.800123	0.846297
20	0.556657	0.623939	0.686540	0.744012	0.795963	0.842073
22	0.553238	0.620157	0.682450	0.739683	0.791483	0.837553
24	0.549546	0.616080	0.678054	0.735049	0.786709	0.832766
26	0.545591	0.611725	0.673372	0.730130	0.781667	0.827743
28	0.541389	0.607107	0.668422	0.724951	0.776384	0.822510
30	0.536953	0.602244	0.663225	0.719533	0.770883	0.817093
32	0.532297	0.597153	0.657801	0.713900	0.765190	0.811517
34	0.527437	0.591851	0.652170	0.708073	0.759326	0.805804
36	0.522388	0.586356	0.646351	0.702074	0.753314	0.799976
38	0.517165	0.580687	0.640365	0.695923	0.747177	0.794052
40	0.511786	0.574862	0.634231	0.689642	0.740932	0.788051
42	0.506266	0.568898	0.627970	0.683251	0.734602	0.781992
44	0.500622	0.562815	0.621600	0.676769	0.728203	0.775891
46	0.494873	0.556632	0.615142	0.670217	0.721756	0.769764
48	0.489034	0.550366	0.608615	0.663613	0.715277	0.763627
50	0.483125	0.544038	0.602038	0.656976	0.708785	0.757496
52	0.477164	0.537668	0.595432	0.650326	0.702298	0.751385
54	0.471170	0.531275	0.588817	0.643682	0.695832	0.745310
56	0.465163	0.524879	0.582212	0.637064	0.689405	0.739286
58	0.459163	0.518502	0.575640	0.630491	0.683037	0.733329
60	0.453192	0.512167	0.569122	0.623985	0.676745	0.727455
62	0.447272	0.505895	0.562680	0.617567	0.670549	0.721680
64	0.441428	0.499711	0.556339	0.611258	0.664469	0.716024
66	0.435683	0.493642	0.550124	0.605085	0.658528	0.710504
68	0.430065	0.487715	0.544062	0.599072	0.652749	0.705142
70	0.424604	0.481959	0.538183	0.593247	0.647159	0.699961
72	0.419332	0.476408	0.532519	0.587641	0.641784	0.694985
74	0.414284	0.471098	0.527106	0.582290	0.636659	0.690244
76	0.409500	0.466070	0.521985	0.577231	0.631818	0.685770
78	0.405026	0.461371	0.517202	0.572511	0.627303	0.681601
80	0.400915	0.457055	0.512813	0.568181	0.623166	0.677782
82	0.397229	0.453189	0.508883	0.564307	0.619464	0.674368
84	0.394049	0.449853	0.505494	0.560967	0.616276	0.671427
86	0.391477	0.447157	0.502754	0.558268	0.613700	0.669053
88	0.389662	0.445255	0.500823	0.556366	0.611884	0.667379
90	0.388889	0.444444	0.500000	0.555556	0.611111	0.666667
	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	$\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$
5	0.572487	0.641567	0.705765	0.764592	0.817600	0.864388
15	0.563926	0.632010	0.695307	0.753346	0.805703	0.852010
25	0.547600	0.613936	0.675748	0.732623	0.784220	0.830282
35	0.524935	0.589127	0.649283	0.705094	0.756337	0.802903
45	0.497760	0.559735	0.618381	0.673501	0.724985	0.772830
55	0.468167	0.528076	0.585512	0.640369	0.692612	0.742291
65	0.438541	0.496661	0.553214	0.608153	0.661480	0.713246
75	0.411857	0.468546	0.524506	0.579721	0.634200	0.687972
85	0.392679	0.448417	0.504034	0.559529	0.614903	0.670162

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$$\Lambda_0(\varphi|\alpha) = \frac{F(\varphi|90^\circ - \alpha)}{K'(\alpha)} + \frac{2}{\pi} K(\alpha) Z(\varphi|90^\circ - \alpha)$$

$$= \frac{2}{\pi} \{K(\alpha) E(\varphi|90^\circ - \alpha) - [K(\alpha) - E(\alpha)] F(\varphi|90^\circ - \alpha)\}$$

$\alpha \backslash \varphi$	65°	70°	75°	80°	85°	90°
0°	0.906308	0.939693	0.965926	0.984808	0.996195	1
2	0.906032	0.939407	0.965633	0.984511	0.995903	1
4	0.905210	0.938559	0.964769	0.983652	0.995130	1
6	0.903857	0.937172	0.963376	0.982315	0.994063	1
8	0.901997	0.935282	0.961512	0.980599	0.992833	1
10	0.899660	0.932934	0.959244	0.978597	0.991511	1
12	0.896881	0.930177	0.956638	0.976384	0.990135	1
14	0.893699	0.927061	0.953755	0.974016	0.988727	1
16	0.890152	0.923634	0.950646	0.971534	0.987299	1
18	0.886280	0.919940	0.947355	0.968969	0.985858	1
20	0.882119	0.916018	0.943918	0.966343	0.984410	1
22	0.877704	0.911904	0.940364	0.963671	0.982958	1
24	0.873068	0.907630	0.936718	0.960968	0.981506	1
26	0.868240	0.903221	0.933000	0.958241	0.980054	1
28	0.863249	0.898703	0.929226	0.955500	0.978604	1
30	0.858117	0.894095	0.925409	0.952751	0.977159	1
32	0.852869	0.889416	0.921563	0.949998	0.975719	1
34	0.847523	0.884681	0.917695	0.947247	0.974286	1
36	0.842100	0.879904	0.913817	0.944502	0.972861	1
38	0.836615	0.875099	0.909935	0.941766	0.971445	1
40	0.831085	0.870277	0.906056	0.939042	0.970039	1
42	0.825524	0.865449	0.902188	0.936335	0.968644	1
44	0.819946	0.860625	0.898337	0.933647	0.967262	1
46	0.814365	0.855814	0.894508	0.930981	0.965894	1
48	0.808792	0.851026	0.890708	0.928341	0.964540	1
50	0.803241	0.846269	0.886942	0.925731	0.963204	1
52	0.797724	0.841553	0.883216	0.923152	0.961885	1
54	0.792252	0.836887	0.879537	0.920610	0.960586	1
56	0.786839	0.832280	0.875911	0.918108	0.959309	1
58	0.781496	0.827742	0.872345	0.915649	0.958055	1
60	0.776237	0.823283	0.868846	0.913240	0.956826	1
62	0.771077	0.818913	0.865421	0.910884	0.955626	1
64	0.766029	0.814645	0.862080	0.908588	0.954457	1
66	0.761110	0.810490	0.858831	0.906357	0.953321	1
68	0.756338	0.806464	0.855685	0.904198	0.952223	1
70	0.751731	0.802581	0.852654	0.902119	0.951166	1
72	0.747312	0.798860	0.849751	0.900129	0.950154	1
74	0.743104	0.795319	0.846990	0.898237	0.949193	1
76	0.739137	0.791983	0.844390	0.896456	0.948288	1
78	0.735442	0.788877	0.841972	0.894800	0.947446	1
80	0.732059	0.786036	0.839759	0.893286	0.946677	1
82	0.729036	0.783497	0.837783	0.891933	0.945990	1
84	0.726434	0.781312	0.836083	0.890770	0.945400	1
86	0.724333	0.779549	0.834711	0.889831	0.944923	1
88	0.722852	0.778307	0.833745	0.889170	0.944587	1
90	0.722222 $\left[\begin{smallmatrix} (-4)1 \\ 6 \end{smallmatrix} \right]$	0.777778 $\left[\begin{smallmatrix} (-5)9 \\ 6 \end{smallmatrix} \right]$	0.833333 $\left[\begin{smallmatrix} (-5)7 \\ 6 \end{smallmatrix} \right]$	0.888889 $\left[\begin{smallmatrix} (-5)5 \\ 5 \end{smallmatrix} \right]$	0.944444 $\left[\begin{smallmatrix} (-5)2 \\ 5 \end{smallmatrix} \right]$	1
5	0.904599	0.937930	0.964135	0.983037	0.994624	1
15	0.891969	0.925384	0.952226	0.972787	0.988015	1
25	0.879076	0.905441	0.934867	0.959607	0.980779	1
35	0.844820	0.882297	0.915757	0.945873	0.973573	1
45	0.817155	0.858217	0.896419	0.932311	0.966576	1
55	0.789537	0.834576	0.877717	0.919353	0.959944	1
65	0.763552	0.812552	0.860443	0.907464	0.953885	1
75	0.741089	0.793624	0.845669	0.897332	0.948733	1
85	0.725315	0.780373	0.835352	0.890270	0.945145	1